Apparent Thermal Emissivity from Surfaces with Multiple V-Shaped Grooves

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A requirement of the advancement in space and high-temperature technology is the transfer of heat by thermal radiation. The efficient transfer of heat by radiation can be accomplished by surfaces of high thermal emissivity. Such surfaces are difficult to obtain, particularly at very high temperatures. This paper presents certain experimental and analytical results on metallic surfaces with multiple V-shaped grooves. The radiant interchange within an infinitely long V-shaped groove is studied, and its apparent thermal emissivity is found as a function of the included angle and the emissivity of the surfaces including the angle. The view factor and the apparent emissivity are plotted for various angles vs the surface emissivity. The experimental results for a 30° included angle are in agreement with the analysis. It is found that, with an included angle of 30° and surface emissivity of 0.3, an apparent emissivity of 0.64 can be obtained. Experimental measurements of the total thermal emissivity of a ground molybdenum surface at various temperatures also are presented. A discussion on actual application of grooves on radiating surfaces and the effects of imperfections present in the grooves due to cutting processes is given.

RECENT advancements in high-temperature vacuum technology and the advent of auxiliary power system requirements in space vehicles[1] quite often have necessitated the use of high-temperature radiators for transferring heat. Strict vacuum requirements rule out the use of oxide coatings at temperatures above 1700°C because their vapor pressure becomes noticeable above these temperatures. If the vacuum requirement is dropped, even then loss of oxide due to vaporization will deem its use for long periods of operation impractical, particularly with the possibility of change of the apparent emissivity with change in oxide coating thickness. It is desirable, therefore, to have a material that can withstand high temperatures while exhibiting high emissivity. A grooved surface of a refractory metal appears to make an excellent choice for such applications.

It has been known that a rough surface has a higher emissivity than a polished one made of the same material. If the geometrical shape of the roughness is regular and known, then an analytical prediction of the emissivity may be possible. V-shaped grooves have a convenient geometric configuration, and they can be applied on metallic surfaces relatively easily, assuming that the angle is not too small.

The radiant interchange within an infinitely long V-shaped groove is studied, and its apparent thermal emissivity is found as a function of the included angle (θ) and the emissivity at the surfaces including the angle. The bounding surfaces of the groove are assumed isothermal and radiate in a gray diffuse manner.[2]

The problem is analyzed by applying radiant flux balances between the surfaces having a view factor F and by estimating the heat radiated to space as a function of F and surface emissivity ϵ. The view factor is found by consideration of infinitesimal area elements and integration of the resulting relationship.

The view factor F and the apparent emissivity ϵ_a are plotted for various angles and ϵ. The analytical results for a groove of 30° included angle are compared with experimental data on molybdenum. It has been found that, for $\theta = 30°$ and $\epsilon = 0.3$, an $\epsilon_a = 0.64$ can be attained, thus effectively more than doubling the radiated heat from an equivalent surface equal to the width of the groove.

Experimental measurements of the total thermal emissivity of a ground molybdenum surface at various temperatures also are presented in this paper.

A discussion follows on actual application of grooves on radiating surfaces and the effects of the imperfections present on the grooves due to the cutting process.

Analysis

The determination of the radiant interchange between gray surfaces is a fundamental problem in heat transfer by radiation. This problem occurs in a wide range of engineering applications and has created the need for a calculation procedure that is general and easy to apply. To achieve a relatively easy formulation, it is necessary to make the following assumptions[3, 4]: 1) the reflection and emission from a surface is of diffuse type; 2) the energy emitted from a surface is uniform through all parts of the surface; 3) the energy incident on all parts of a surface is uniform; 4) the groove is infinitely long, and the surfaces, including the angle, are of relatively small width; and 5) the groove is symmetrical, and the surfaces are isothermal. The first assumption indicates that the emission and incidence angle at which the energy is emitted or impinging on a surface need not be considered. The second and third assumptions mean that each surface will be treated as a whole, and no consideration will be given to energy transfer from different parts of the surface. The third and fourth eliminate end effects and variation in surface emissivity and heat radiated from the surfaces. There are some arguments against the first three assumptions, i.e., that the reflection varies along the surfaces and that the energy emitted will be greater at the apex of the groove. Consideration of these arguments results in integral solutions that have to be solved numerically; a simplified method is used here. Further justification will be given in subsequent sections.

Radiation Exchange

Consider an infinitely long groove with an included angle θ. Assume the surfaces to be gray with uniform emissivity ϵ and at uniform temperature.

Let Q_b be the blackbody radiation emanating from a surface equal in size to surface A (see Fig. 1). Then the radi-

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Fig. 1 Idealized V-shaped surface groove.

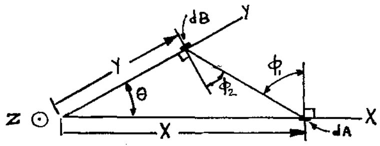


Fig. 2 Mathematical model of V-groove.

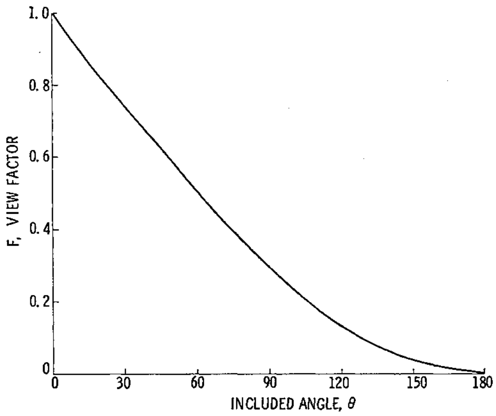


Fig. 3 Variation of view factor with included angle.

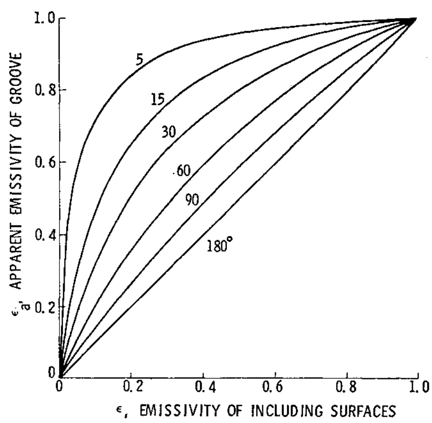


Fig. 4 Variation of apparent emissivity with surface emissivity at various included angles.

ation emitted from A is $Q_1 = Q_b\epsilon$. Of this, $Q_2 = FQ_1$ strikes surface B, where F is the view factor of the surfaces A and B since, from the reciprocity principle, $F_{AB} = F_{BA} = F$ for surface $A = B$. The portion of radiation $Q_3 = (1 - F)Q_1$ goes to space. Surface B reflects $Q_4 = (1 - \epsilon)Q_2$ since, by definition and Kirchhoff's law of radiation, $\epsilon = \alpha = 1 - \rho$, where α and ρ are the absorptivity and reflectivity of the surfaces, respectively.

The amount $Q_5 = (1 - F)Q_4$ goes to space, and $Q_6 = FQ_4$ strikes surface A. Surface A, in turn, reflects $Q_7 = (1 - \epsilon) Q_6$, of which $Q_9 = (1 - F)Q_7$ goes to space and $Q_8 = FQ_7$ strikes surface B. Carrying this reasoning further and adding all terms that surface A radiates to space yields

$$\begin{aligned} Q_{\text{space}} &= Q_3 + Q_5 + \ldots + Q_n \\ &= (1 - F)Q_1 + (1 - F)(1 - \epsilon)FQ_1 + \\ &\qquad (1 - F)(1 - \epsilon)\, F(1 - \epsilon)FQ_1 + \ldots \\ &= (1 - F)Q_1[1 + F(1 - \epsilon) + F^2(1 - \epsilon)^2 + \\ &\qquad F^3(1 - \epsilon)^3 + \ldots + F^n(1 - \epsilon)^n] \end{aligned}$$

Let $x = F(1 - \epsilon)$; then

$$Q_s = (1 - F)Q_1[1 + x + x^2 + x^3 = \ldots + x^n]$$

power series. Since $F(1 - \epsilon) < 1$, then $x < 1$, and the foregoing series converges to $1/(1 - x)$ as $n \to +\infty$; therefore

$$Q_s = \frac{(1 - F)Q_1}{1 - F(1 - \epsilon)} = \frac{(1 - F)\epsilon Q_b}{1 - F(1 - \epsilon)} \qquad (1)$$

If the apparent emissivity ϵ_a of the groove is defined as the ratio of the heat emitted to space by the groove over the heat emitted by a blackbody of area equal to the projection of the groove, then

$$\epsilon_a = \frac{Q_s}{Q_b \sin\theta/2} = \left(\frac{\epsilon}{\sin\theta/2}\right)\left[\frac{1 - F}{1 - F(1 - \epsilon)}\right] \qquad (2)$$

To obtain ϵ_a as a function of the included angle and ϵ only, the view factor F of the surfaces A and B must be found.

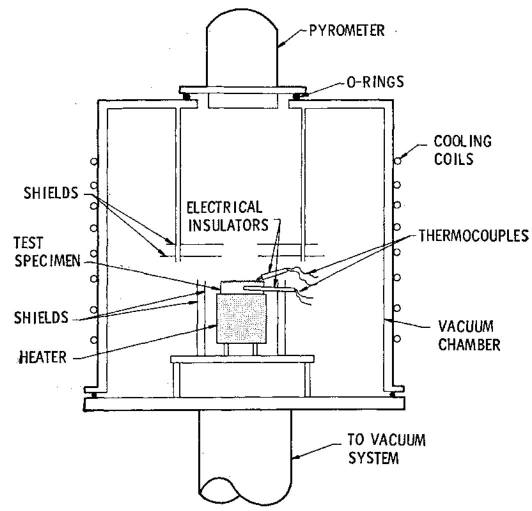


Fig. 5 Schematic setup of experimental apparatus.

View Factor

The view factor of two surfaces is given by the integral

$$F = \frac{1}{\pi A}\int_A \int_B \frac{\cos\phi_1 \cos\phi_2}{r^2}\, dA dB$$

For the geometry of the groove, the integral is set up as follows (see Fig. 2):

$$\begin{aligned} dA &= dx dz_1 \\ dB &= dy dz_2 \\ r^2 &= x^2 + y^2 - 2xy\cos\theta + (z_1 - z_2)^2 \\ \cos\phi_1 &= (x/r)\sin\theta \\ \cos\phi_2 &= (y/r)\sin\theta \end{aligned}$$

and

$$F = \frac{1}{\pi l x} \times \int_0^x \int_0^y \int_{-l}^{+l} \int_{-l}^{+l} \frac{xy \sin^2\theta\, dx dy dz_1 dz_2}{[x^2 + y^2 + (z_1 - z_2)^2 - 2xy\cos\theta]^2}$$

Carrying out the two first integrations using the initial assumption that the groove is of infinite length ($-\infty \leq l \leq +\infty$), the view factor becomes

$$F = \frac{1}{8}\int_0^x \int_0^y \frac{\sin^2\theta\, dx dy}{(x^2 + y^2 - 2xy\cos\theta)^{3/2}}$$

Integrating further, with $x = y = 1$, yields

$$F = 1 - \left(\frac{1 - \cos\theta}{2}\right)^{1/2} = 1 - \sin\frac{\theta}{2} \qquad (3)$$

This relationship is shown in Fig. 1. It can be seen that for

$\theta = 0$ (parallel plates), $F = 1$; for $\theta = \pi/2$ (perpendicular plates), $F = 0.3$; and for $\theta = \pi$ (flat plate), $F = 0$, which is correct.

Introducing Eq. (3) for F into Eq. (2), the apparent emissivity of the groove as a function of θ and ϵ is obtained as

$$\epsilon_a = [1 - (1/\epsilon - 1)\sin\theta/2]^{-1} \qquad (4)$$

Analytical Results

The view factor has been plotted for various included angles of V-shaped grooves using Eq. (3). The plot is shown in Fig. 3. It is seen there that the values of F at 0° and 180° are those that are expected, and at 90° they checked with McAdams' value of 0.3.

In Fig. 4, the apparent emissivity (ϵ_a) is plotted against the surface emissivity (ϵ) for five different values of included angles. There, it is seen that the apparent emissivity increases somewhat at $\theta = 90°$. The increase becomes quite pronounced as the included angle becomes smaller. It is apparent also that the effect of the groove is significant for values of ϵ less than 0.6; above this value, the increase of the apparent emissivity tapers off, as it should, since, when $\epsilon = 1$, the ϵ_a also must be equal to 1. As an example, if $\epsilon = 0.2$ for a particular material, by machining grooves on that surface with $\theta = 30°$, the apparent emissivity becomes $\epsilon_a = 0.5$. As a check of Eq. (4), it can be seen in Fig. 4 that $\epsilon = \epsilon_a$ at $\theta = 180°$.

Experimental Results

The method by which emissivity measurements were made was to measure the true temperature (T) of the heated specimen by a thermocouple and the apparent temperature (T_a) by a total radiation pyrometer calibrated to read blackbody temperature. The pyrometer measures the amount of radiation emitted by the specimen, i.e., it measures $T_a^4 = \epsilon T^4$. Thus, the emissivity can be found from $\epsilon = (T_a/T)^4$.

The apparatus used is shown in Fig. 5. It consists of a shielded electron bombardment heater facing a disk-shaped specimen. This assembly then is placed inside an evacuated chamber. The pyrometer used was Leeds and Northrup 8890-2491 Rayotube with a calcium fluoride window. The instrument was calibrated by means of a carbon specimen that can be considered as a blackbody. The calibration obtained was ±0.5°C in agreement with the manufacturer's indications.

The true temperature was measured by two 0.004-in.-diam Pt-Pt 10% Rh thermocouples. One was imbedded inside a 0.030-in.-diam hole drilled through the specimen to within $\frac{1}{32}$ in. of the test surface, and the other was spot-welded near the test surface. The two thermocouples agreed within 2°C. Therefore, the deviation on the emissivity reading was a fraction of 1%.

One typical specimen tested was a molybdenum disk $\frac{3}{8}$-in. thick and 1.00-in. diam, the surface of which was grooved with 30° included angles 0.015-in. wide, with 0.001-in. flats and 0.005-in. fillets. The grooves were cut by grounding, exhibiting the same grade as the one used for the first specimen. This technique was used for obtaining the same finish of the walls of the surfaces, including the angle, as that of the first specimen. For removal of surface impurities, the specimens were cleaned in tetrachloride, ultrasonically, and then were fired in vacuum at 1500°C. To obtain various emissivities for the same specimens, the emissivity was measured at various temperatures. The results of the measurements are shown in Fig. 6 and compared with the analysis on Fig. 7.

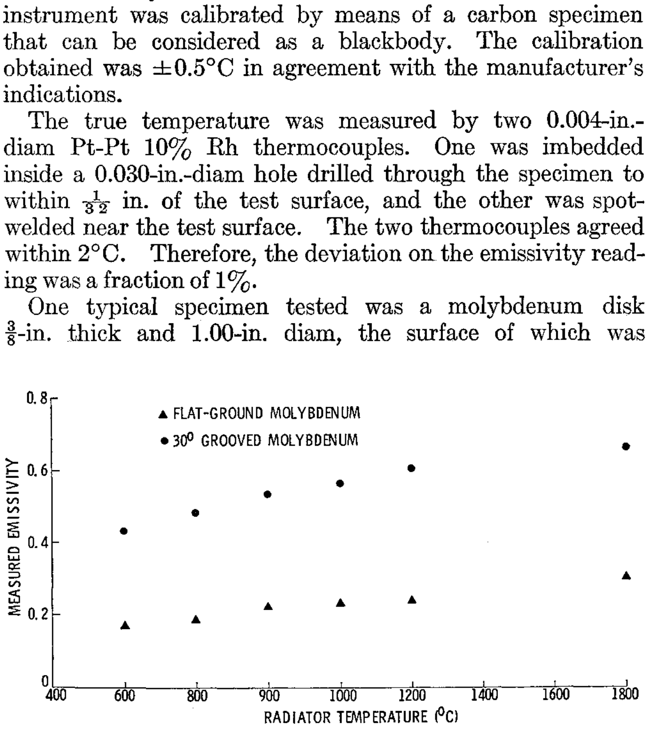


Fig. 6 Measured emissivity of grooved and flat-ground molybdenum at various specimen temperatures.

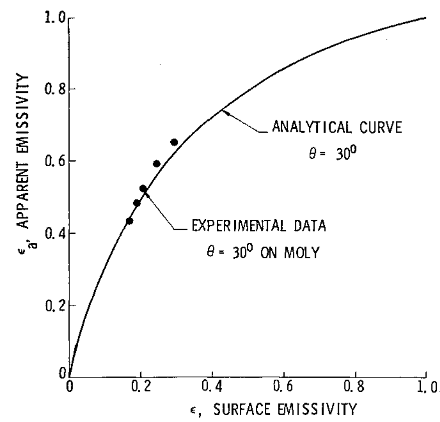


Fig. 7 Comparison of analytical and experimental measurement of variation of apparent emissivity with surface emissivity for 30° included angle.

Discussion

The agreement of experimental data with the analytical results shown in Fig. 7 is very good. The presented data definitely are only a small part of the emissivity region of which one might wish to have experimental verification, but they cover the surface emissivities that most refractory metals exhibit. Therefore, the result is of practical importance.

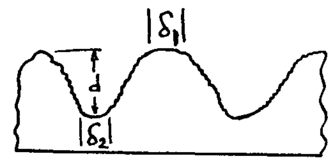
Furthermore, discussing the validity of the assumptions made in the analysis, let us investigate a little more closely the actual surface of a groove cut by a machine tool. Figure 8 shows a blown-up view of a grooved surface.

The surface of the groove quite possibly could be rougher than anticipated; secondly, it is very hard to cut sharp successive corners and, particularly, sharp apexes. If δ_1 and δ_2 represent the width of the flat little areas as shown in Fig. 8, and if $\delta_2 \ll \delta_1$ and $\delta_2 \approx 0$, then the heat radiated from the disk will be

$$Q < \epsilon\Sigma\delta_{10}\, l_i + \epsilon_a[(\pi D^2/4) - \Sigma\delta_{10}\, l_i]\sigma T^4$$

When δ_1 approaches zero, then $Q = (\epsilon_a\sigma T^4)\pi D^2/4$. If δ_1 and δ_2 are significantly large, then it becomes difficult to estimate Q because the shape of the groove will be that of a trapezoidal channel that requires an additional analysis. It was mentioned previously that the surface most probably will be rougher than anticipated; therefore, its emissivity will be higher than ϵ. This effect, to a certain degree, should compensate for imperfections at the corners. The experimental results have indicated that, so long as $\delta_1/d \approx \delta_2/d < 10\%$, the heat radiated by the grooved surface can be found directly from the apparent emissivity of the groove multiplied

Fig. 8 Representation of a surface after cutting.



by the radiator area given by its overall dimensions multiplied by σT^4.

In face of these irregularities existing in a machined grooved surface, the error introduced by the first three assumptions will be small for all practical purposes. However, it might become significant at very small angles, i.e., smaller than 10°.[5, 6]

It should be kept in mind that the depth of the grooves should be kept small so that the walls remain isothermal. To make the grooves too small, however, introduces new difficulties. The roundness at the top becomes significant; therefore, the surfaces of the grooves, on the average, are out of flat. Here again, experiments have indicated that, if $(r/d) < 5\%$, the relationships discussed hold well.

In conclusion, it can be stated that the application of V-shaped grooves on a metallic surface will increase its thermal emissivity in a practicable manner.

The assumptions made initially are justified for all practical purposes, considering the difficulties presented in the analysis when assumptions 1–3 are dropped.

It is important to point out that a groove of less than 45° angle provides apparent emissivities two to four times greater than the surface emissivities. For base metals this is quite significant, since most of them exhibit total thermal emissivities of less than 0.2.

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Measurement of Mean Particle Sizes of Sprays from Diffractively Scattered Light

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The angular distribution of scattering for polydispersions of particles distributed according to the Upper Limit Distribution Function is examined and is found to lack the sensitivity necessary to permit determination of size distribution. However, the volume-to-surface mean diameter is found to be directly dependent upon angular distribution of intensity for a wide variety of shapes of the distribution function. Therefore, the combination of both a scattering experiment together with a transmission experiment can be used to obtain both particle concentration and volume-to-surface mean diameter of particles in a spray. While there is no limitation with regard to the maximum diameter, the actual upper size limit that is measurable experimentally is controlled by considerations related to angular resolving power. Experimental results that show agreement between the volume-to-surface mean diameter as determined by scattering experiment and by microscopic examination are given for solid spheres.

THE measurement of particle sizes present in liquid sprays poses a problem of considerable importance that has received frequent attention in the past. In the presence of evaporation of the liquid medium, only the use of high-speed photomicrography to examine the particles in situ is appropriate as a means of determining particle sizes.[1, 2] The method has various difficulties related to the high time resolution required, e.g., the very small depth of focus of the microscope, the small number of particles present within the focal plane, and the question of whether these particles constitute a representative sample. The careful application of the technique undoubtedly yields the most accurate results, and the method may be thought of as the primary standard of measurement in the present instance. On many occasions the awkwardness of application of the primary standard is an overwhelming drawback to its frequent use. It is, therefore, natural to inquire about other methods that might be used to determine the particle sizes present in sprays, methods that, although lacking the precision of the primary standard, do possess the advantages common to secondary methods.

A technique based on the scattering properties of the particles appears to fulfill the forementioned description, and investigation of such a technique first was performed by Chin, Sliepcevich, and Tribus[3, 4] using a theory due to Gumprecht and Sliepcevich[5] which described the scattering properties of a polydispersion. This theory is applicable for polydispersions of such low concentration and/or limited spacial distribution so as to constitute a small optical depth and fur-

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